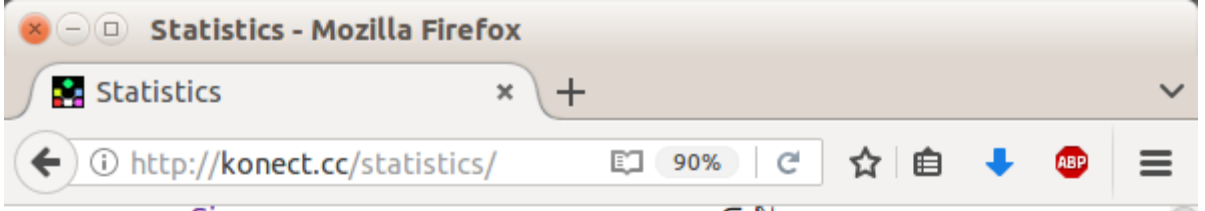
Network Statistics

- A statistic is a real number that characterizes a network
- Examples:
  - Average degree (d)
  - Number of triangles (t)
  - Diameter (δ)
  - Clustering coefficient (c)
  - Gini coefficient of degree distribution (G)
  - Degree assortativity (ρ)

More Statistics

- Number of wegdes (s)
- Number of squares (q)
- Number of claws (z)
- Number of crosses (x)
- Maximum degree (d_{max})
- Relative maximum degree ($d_{MR} = d_{max} / d$)
- Number of degree-1 nodes (d_1)
- 50-percentile effective diameter ($\delta_{0.5}$)
- Relative edge distribution entropy (H_{er})
- Bipartivity ($b_A = 1 – \lambda_{min}[A] / \lambda_{max}[A]$)
- Normalized two-star count ($s_d = s / (n\ d\ (d – 1) / 2)$)
- Eigenvalues of certain matrices ($a = \lambda_2[L]$, $|\lambda_{max}[A]|$, …)
- etc.



n =	Size	$\in \mathbb{N}$
m =	Volume	$\in \mathbb{N}$
$\bar{m}$ =	Unique edge count	$\in \mathbb{N}$
d =	Average degree	$\in \mathbb{R}^+$
d_{max} =	Maximum degree	$\in \mathbb{N}$
p =	Fill	$\in [0, 1]$
$\tilde{m}$ =	Average edge multiplicity	$\in \mathbb{R}^+$
ζ =	Negativity	$\in [0, 1]$
s =	Wedge count	$\in \mathbb{N}$
z =	Claw count	$\in \mathbb{N}$
x =	Cross count	$\in \mathbb{N}$
N =	Size of LCC	$\in \mathbb{N}$
N_s =	Size of LSCC	$\in \mathbb{N}$
ρ =	Degree assortativity	$\in [-1, +1]$
$\rho^{\pm}$ =	In/outdegree correlation	$\in [-1, +1]$
$\|A\|_2$ =	Spectral norm	$\in \mathbb{R}^+$
G =	Gini coefficient	$\in [0, 1]$
γ =	Power law exponent	$\in \mathbb{R}^+$
γ_t =	Tail power law exponent	$\in \mathbb{R}^+$
H_{er} =	Relative edge distribution entropy	$\in [0, 1]$
c =	Clustering coefficient	$\in [0, 1]$
t =	Triangle count	$\in \mathbb{N}$
δ =	Diameter	$\in \mathbb{N}$
$\delta_{0.5}$ =	50-Percentile effective diameter	$\in \mathbb{R}^+$
$\delta_{0.9}$ =	90-Percentile effective diameter	$\in \mathbb{R}^+$
δ_m =	Mean distance	$\in \mathbb{R}^+$
y =	Reciprocity	$\in [0, 1]$
T_4 =	4-Tour count	$\in \mathbb{N}$
q =	Square count	$\in \mathbb{N}$
a =	Algebraic connectivity	$\in \mathbb{R}^+$

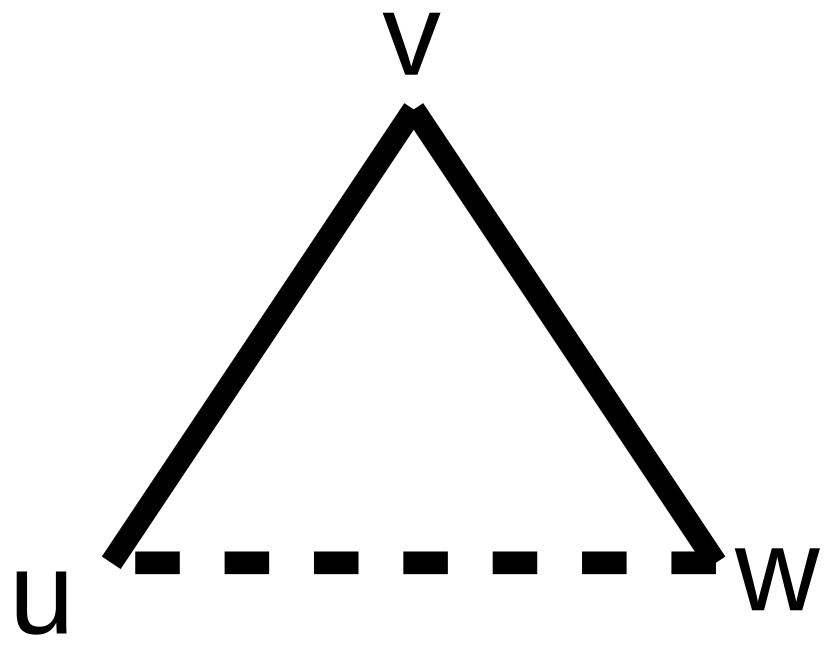
http://konect.cc/statistics/

Clustering Coefficient

$$c = 3t / s$$

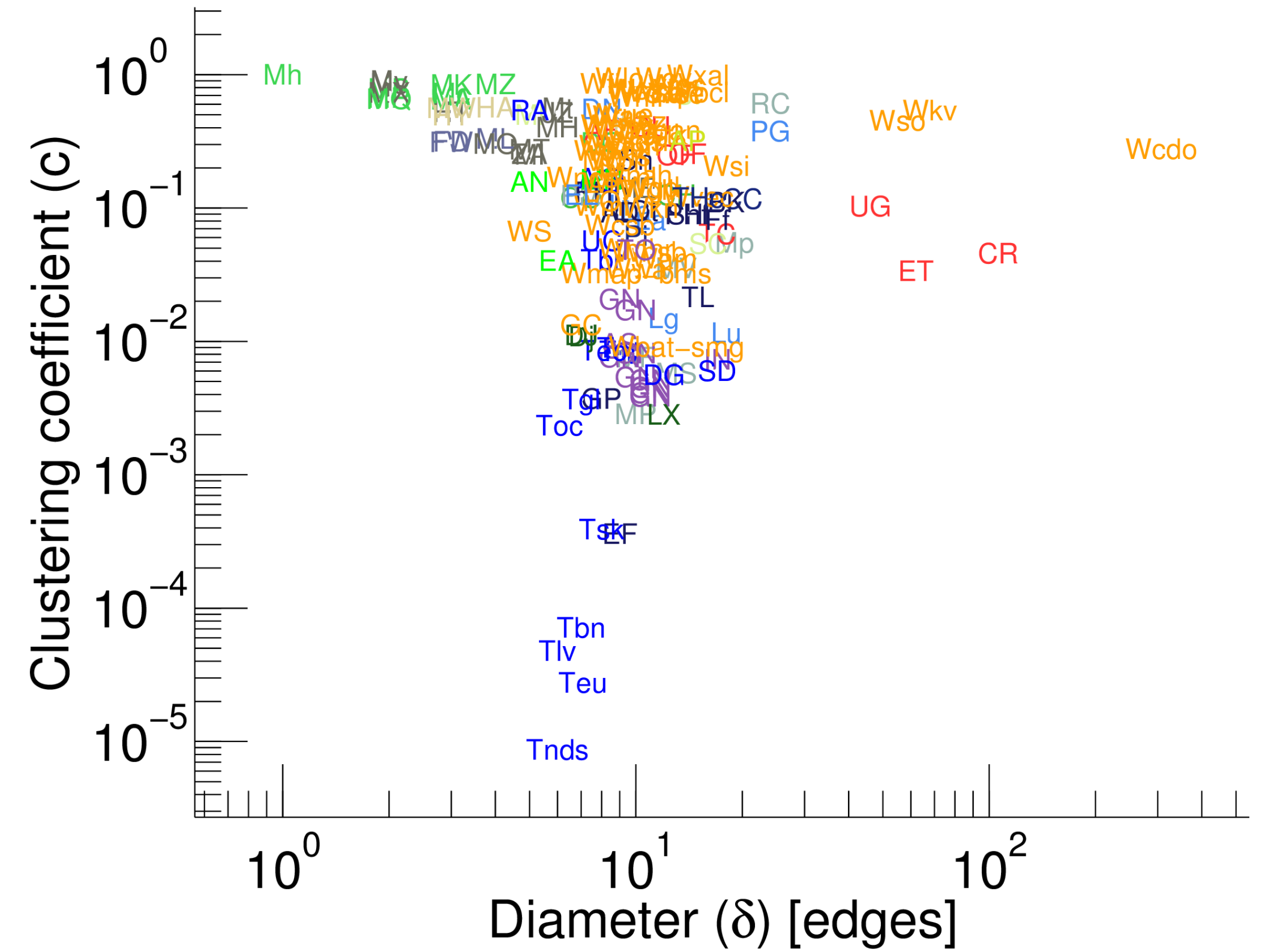
s : number of wedges

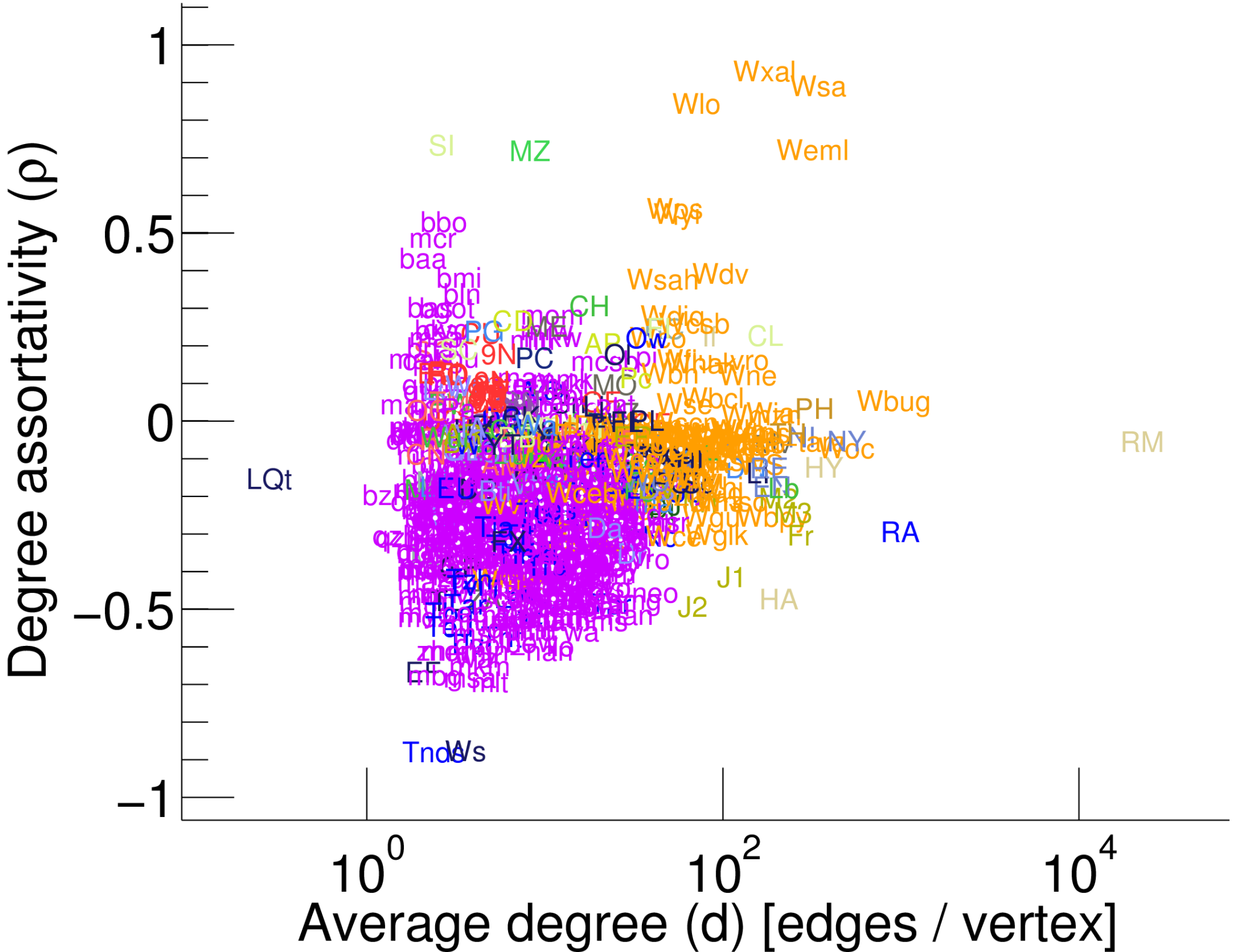
t : number of triangles



$$c = P(uw \mid uv \wedge vw)$$

http://konect.cc/statistics/clusco/


Clustering coefficient (c)
Diameter (δ) [edges]
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^0
10^1
10^2
Mh
MK
MZ
RC
PG
Wso
Wkv
Wcdo
Wsi
UG
CR
ET
WS
EA
TL
Lg
Lu
SD
Toc
Tbn
Tlv
Teu
Tnds


Degree assortativity (ρ)
1
0.5
0
−0.5
−1
Average degree (d) [edges / vertex]
10^0
10^2
10^4
Wxal
Wsa
Wlo
Sl
MZ
Weml
bbo
mcr
baa
bmi
Wsah
Wdv
CH
PC
CL
Wbn
Wne
Wbcl
Wbug
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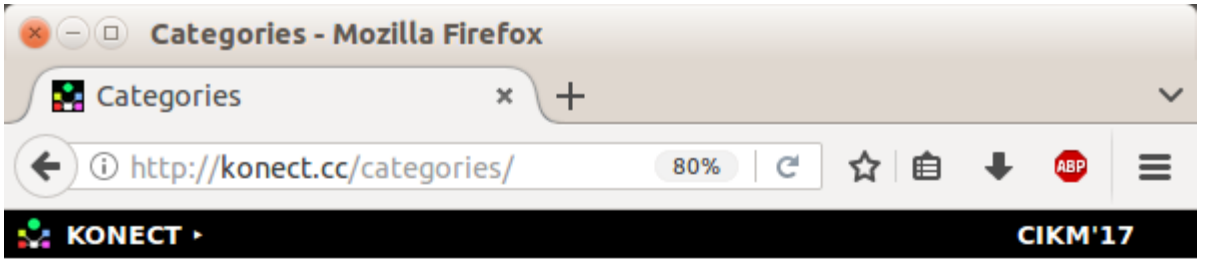
NY
RM
LQt
HY
RA
Fr
J1
HA
J2
Tnde
Ws



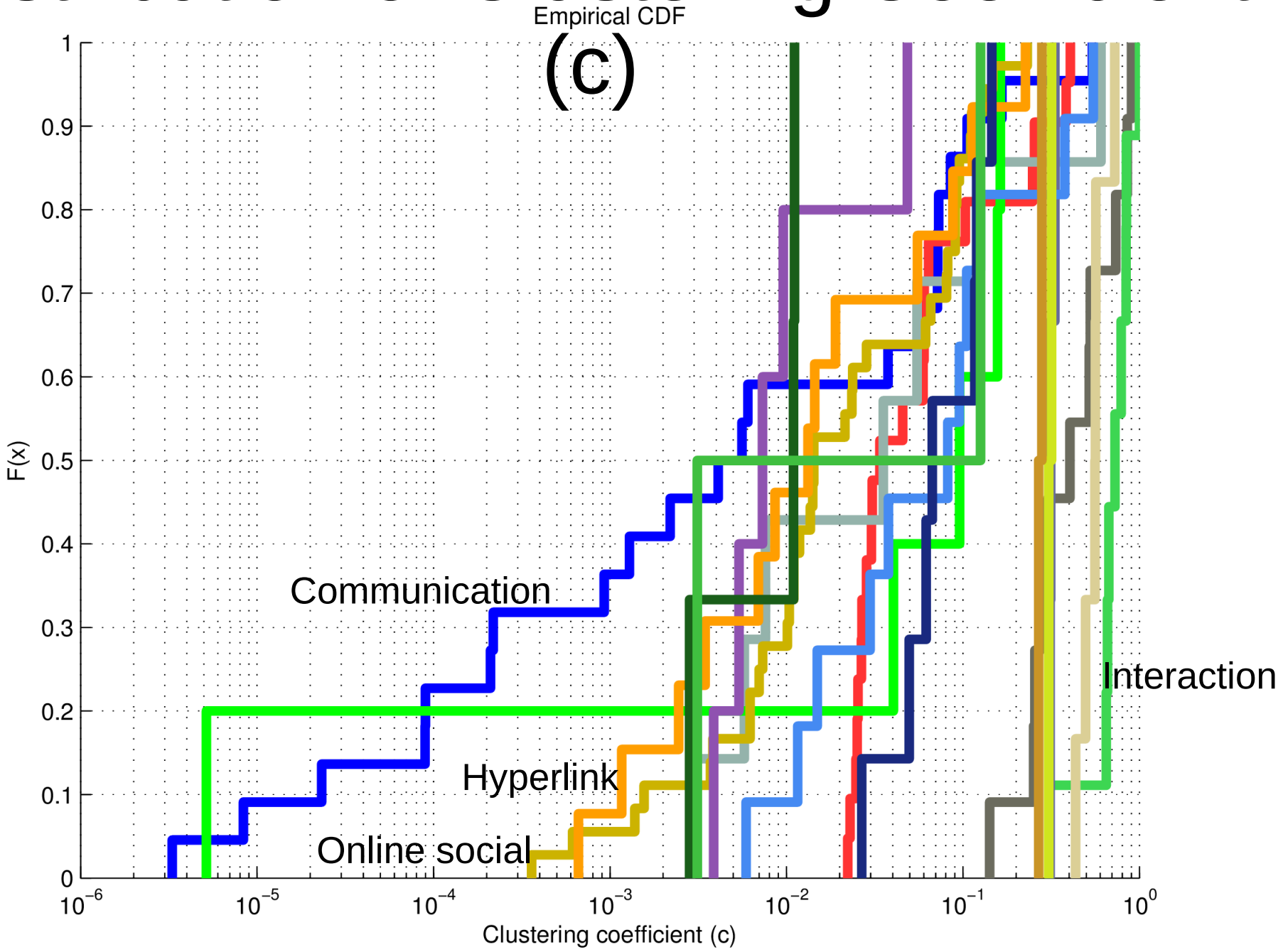
Categories

In KONECT, each network belongs to a **category** which denotes its semantics.
Examples of categories are social networks, road networks, and citation networks.

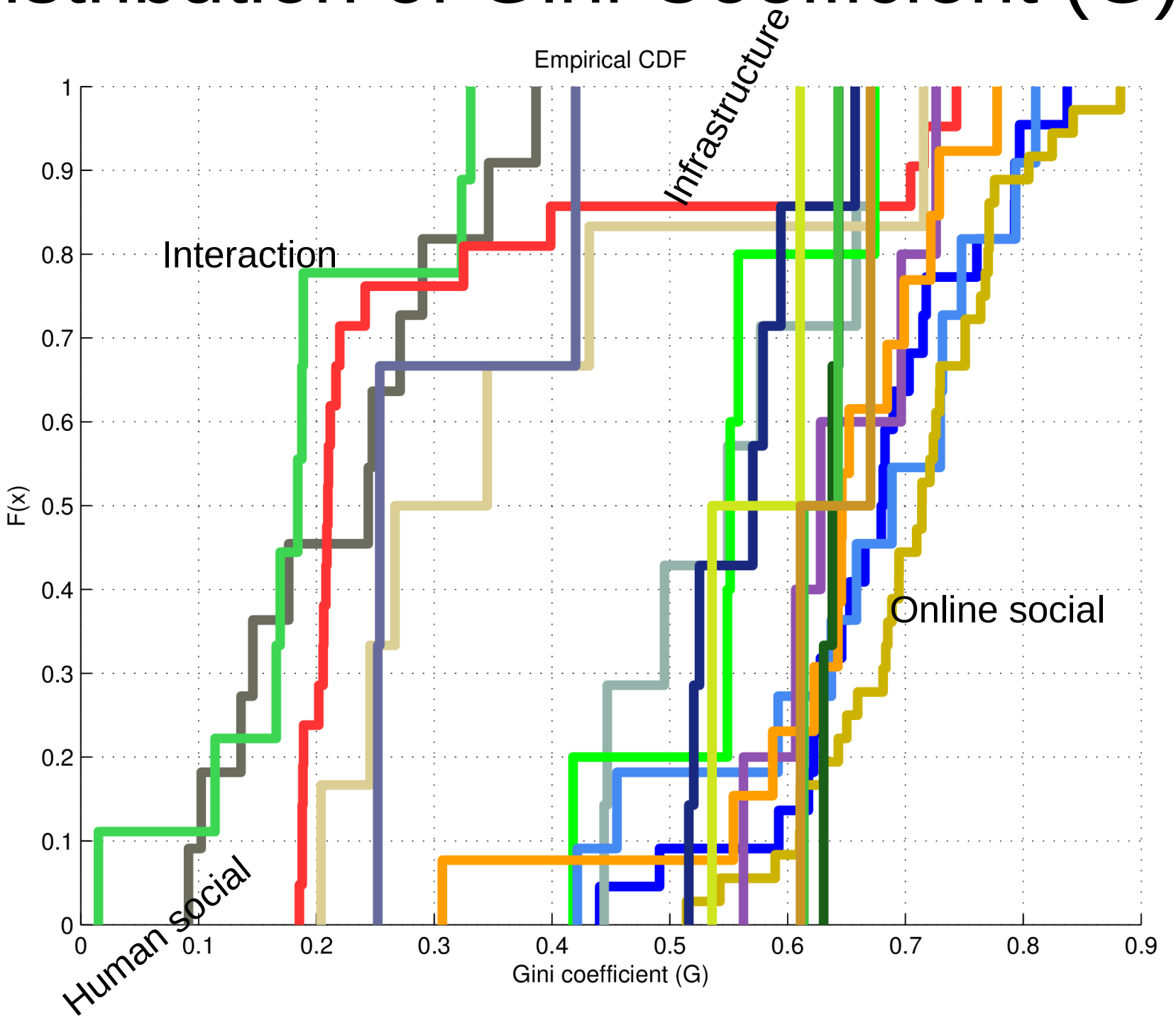
Category	Count
Affiliation	17
Animal	9
Authorship	809
Citation	7
Co-authorship	3
Co-citation	2
Communication	42
Computer	13
Feature	17
Human contact	5
Human social	12
Hyperlink	191
Infrastructure	23
Interaction	25
Lexical	5
Metabolic	7
Miscellaneous	12
Online contact	15
Online social	46
Rating	15
Software	3
Text	10
Trophic	3

http://konect.cc/categories/

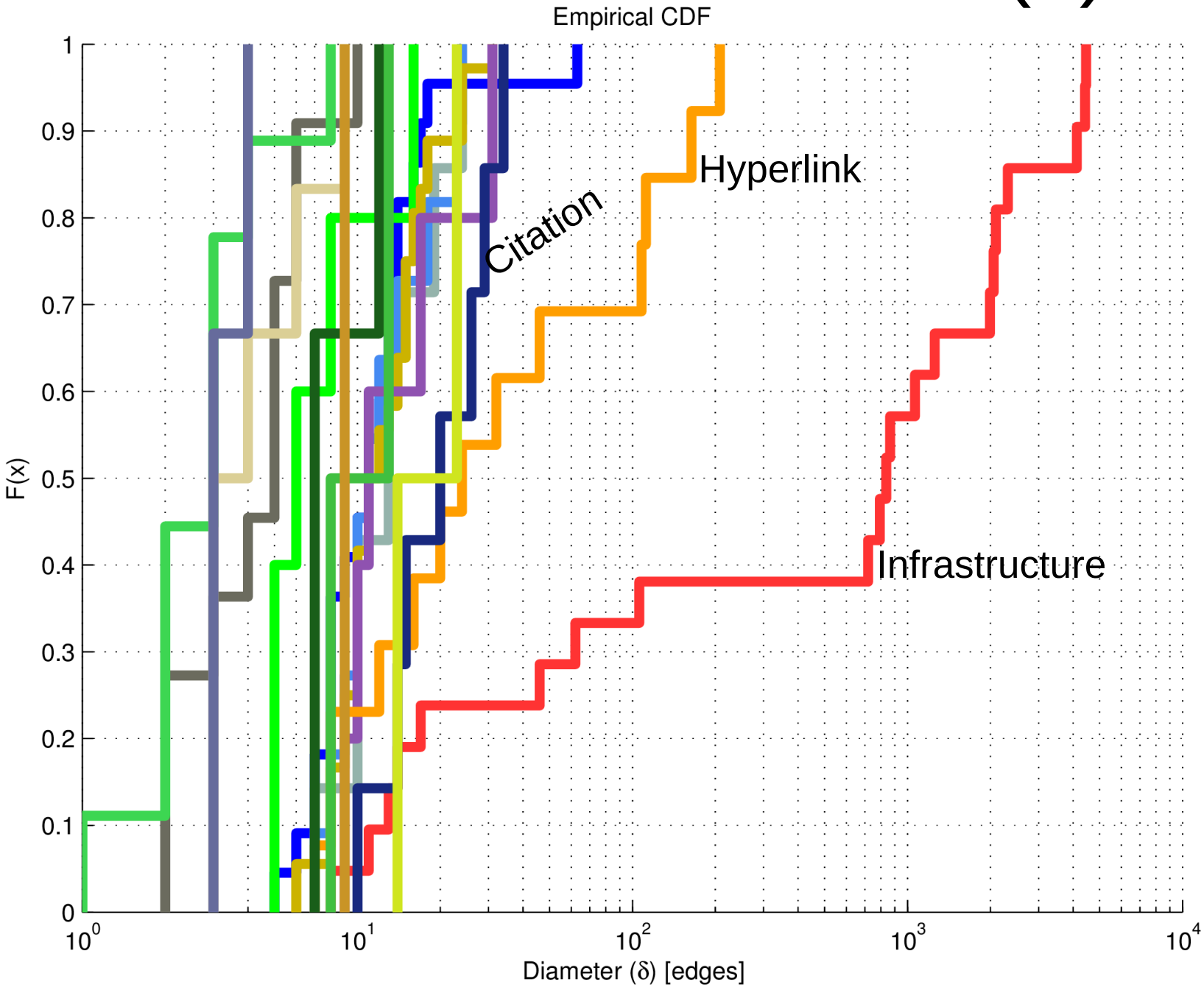
Distribution of Clustering Coefficient



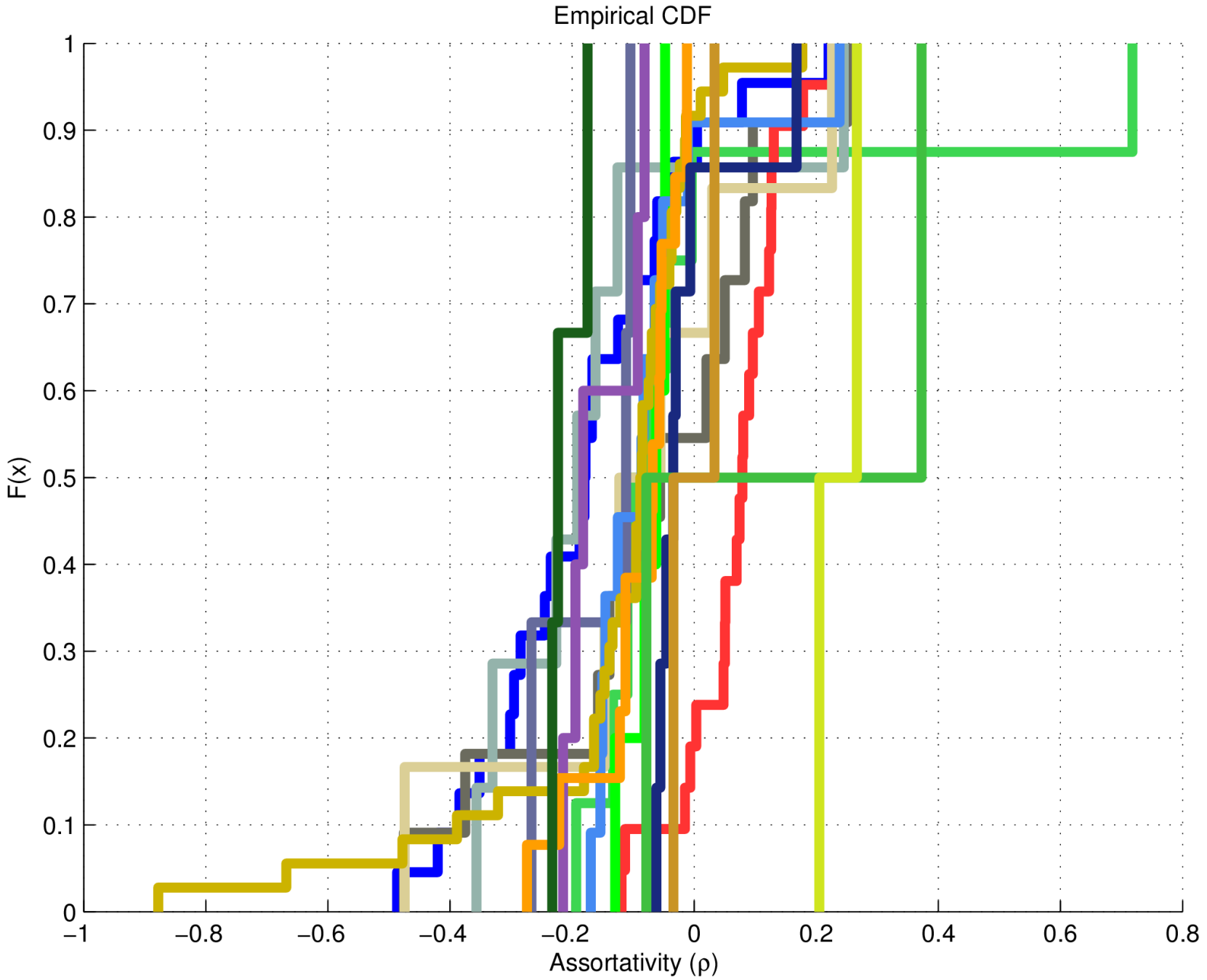
Distribution of Gini Coefficient (G)



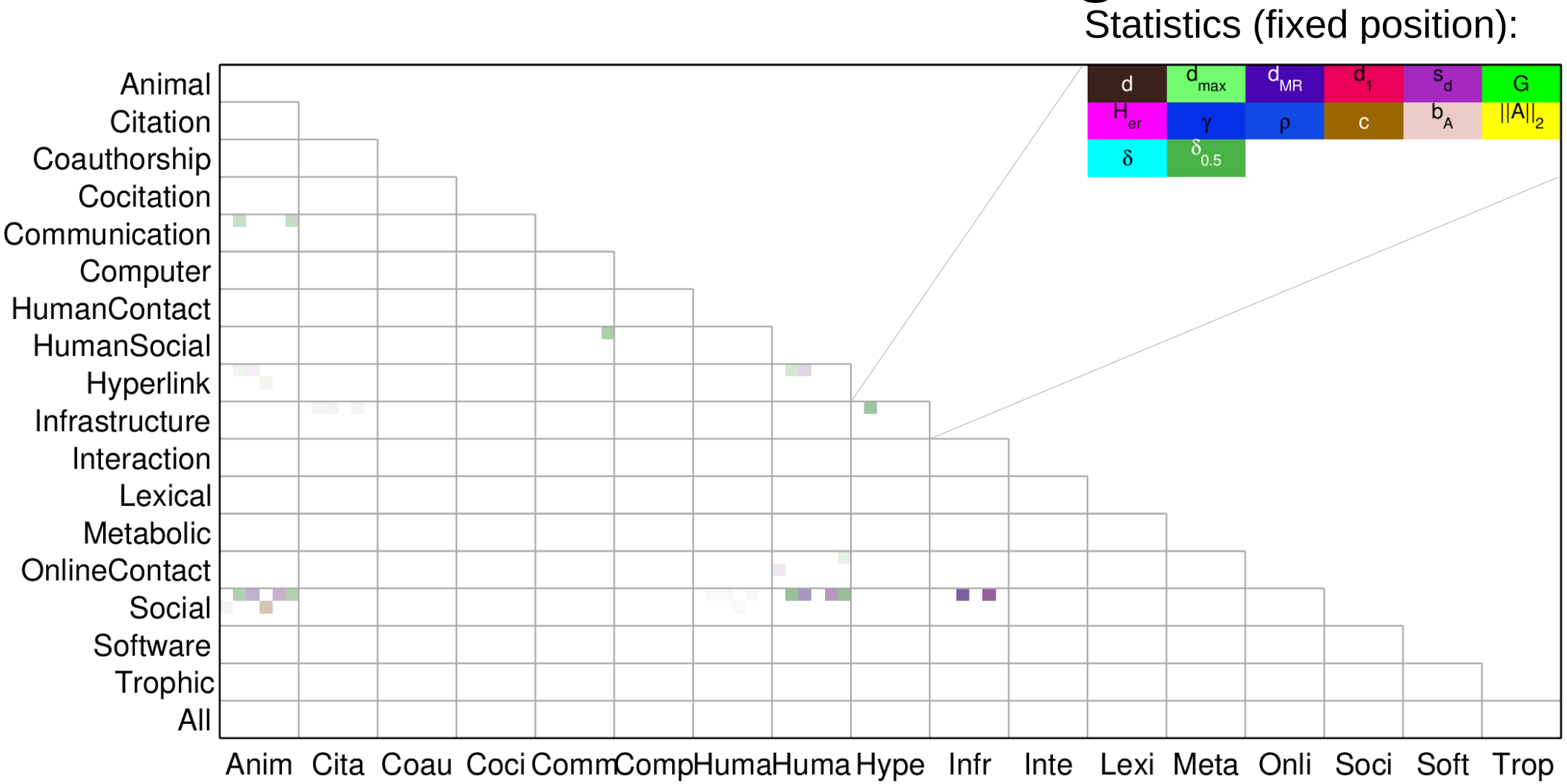
Distribution of Diameter (δ)



Degree Assortativity (ρ)

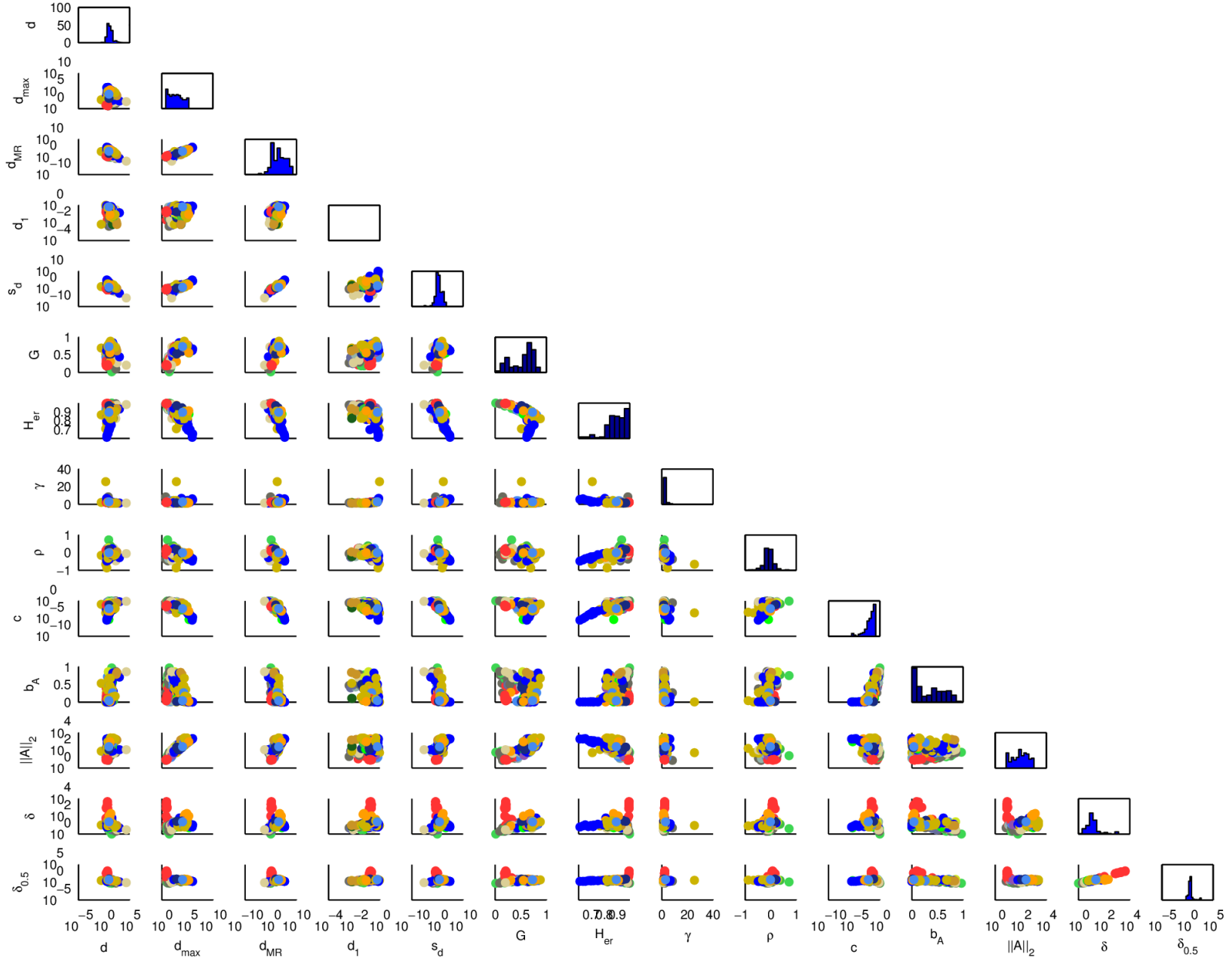


Statistical Testing

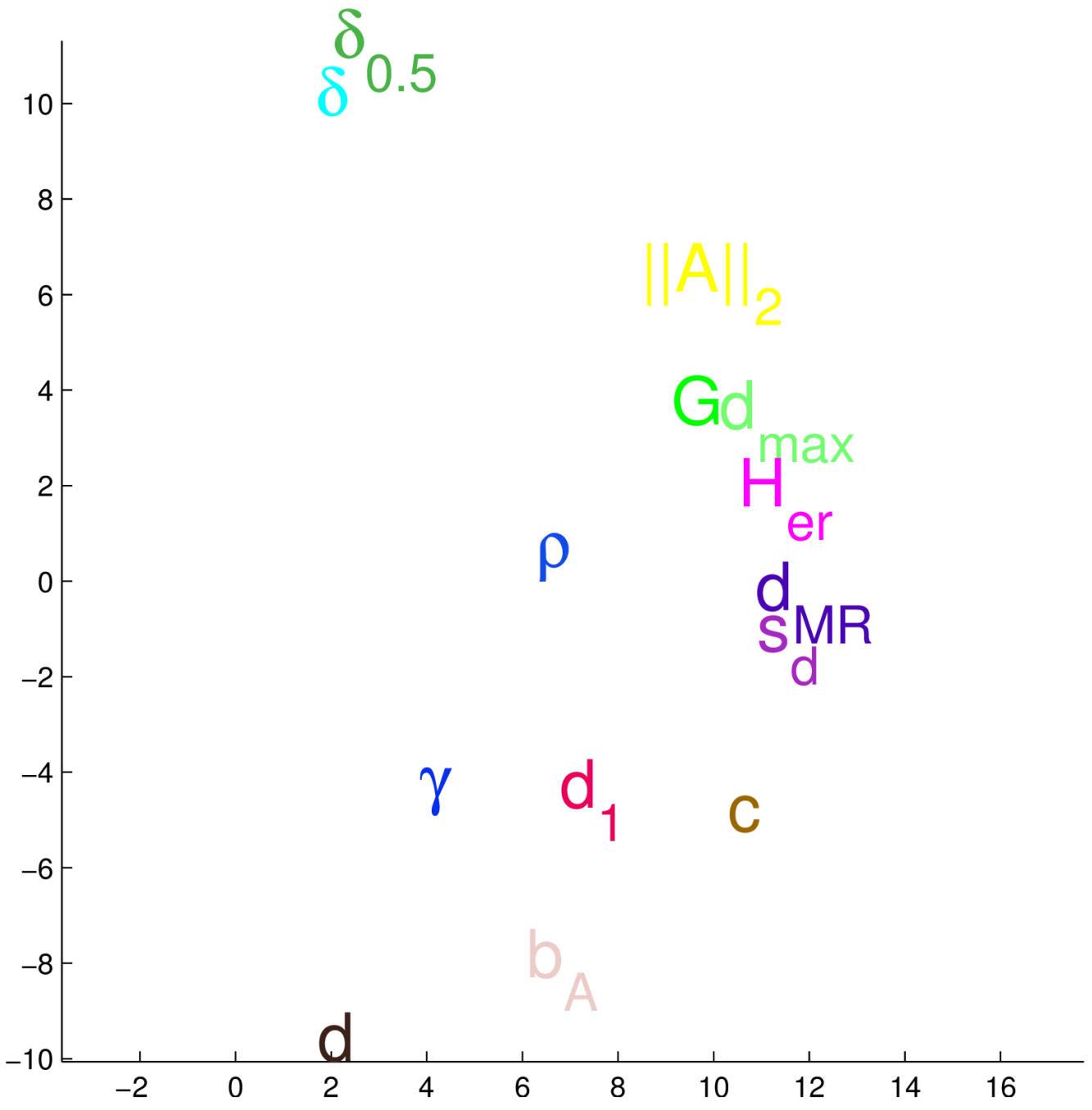


Kolmogorov–Smirnov test on each pair of categories; non-white cell when statistic is significantly different ($p < 0.10$). Base colour by HSL: Hue denotes network statistic; S & L is constant. Shown colour is interpolated between base colour and white for $0 \leq p \leq 0.10$.

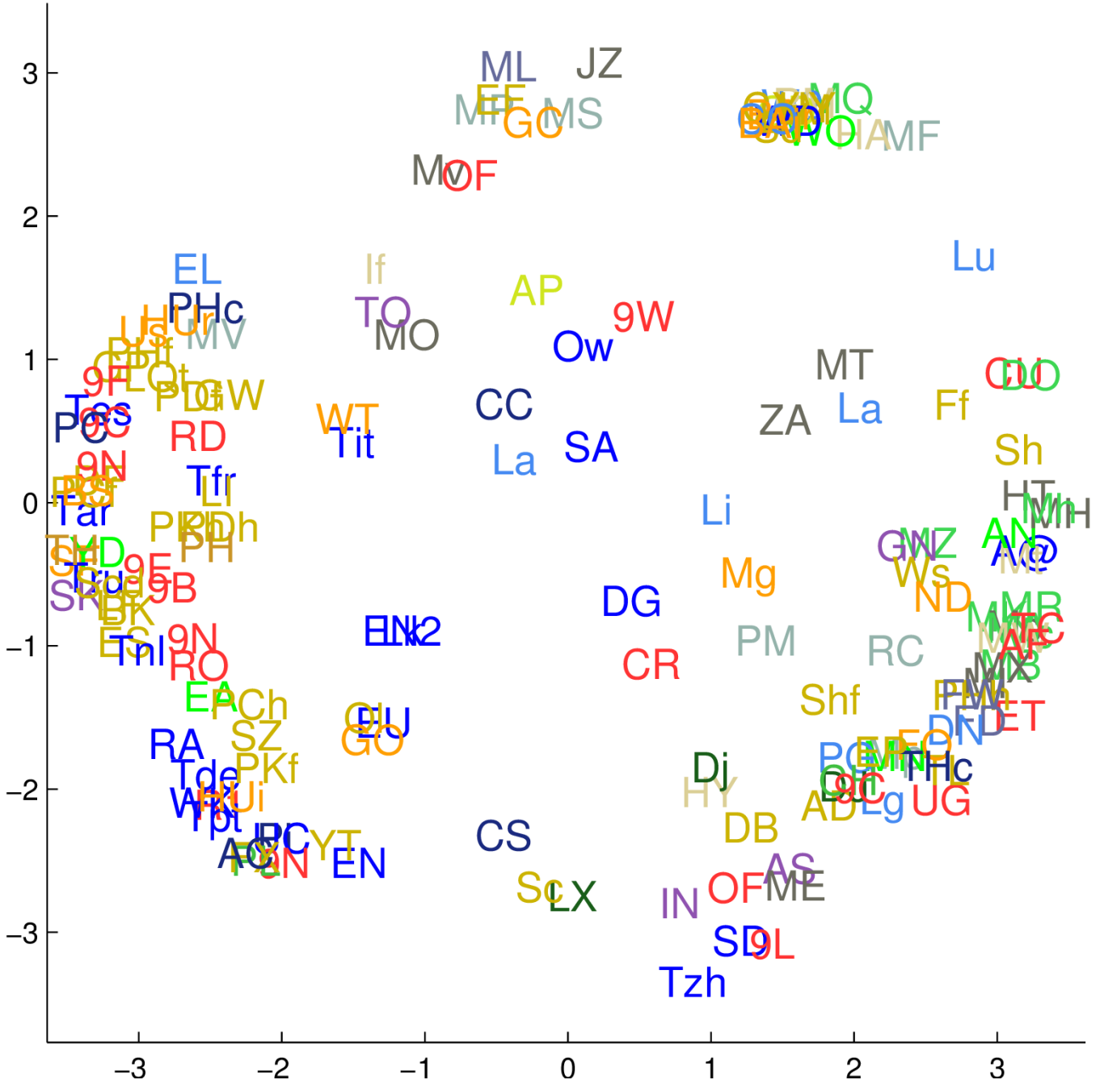
Statistics Are Not Uncorrelated



Principal Component Analysis of Statistics



PCA of Network Datasets



Feature Engineering

- Find size-independent formulations of statistics
  - E.g., c instead of t
- Avoid highly correlated statistics
  - E.g., keep only one of G and P
- Find statistics that are easy to compute
  - E.g., algebraic connectivity (a) needs $O(n^2)$ runtime

Related Work (In Progress)

- Characterizing the structural diversity of complex networks across domains, Kansuke Ikehara, Aaron Clauset

[https://arxiv.org/abs/1710.11304]